APPENDIX A

A PRELIMINARY TEST OF THE DEMAND PROJECTIONS

This appendix presents a comparison of our projected levels of per capita daily demand with the consumption actually observed in communities which neither restricted use nor augmented normal supply sources during the drought. For these communities (9 of the 28 for which projections were made) there is no reason to believe that response to the drought should have interfered with the accurate prediction of actual levels of demand. In Table 51 we summarize the results of this comparison. For each system and year we show the predicted level of per capita daily demand, the 95 percent confidence interval for that prediction, the observed per capita daily consumption and the percentage difference between the projected and observed quantities. This last figure is computed as:

$$\%\ \text{Difference} = \frac{P_{it} - [C_{it}/(N_{it} \cdot 365)]}{P_{it}} \times 100; \qquad \text{(A-1)}$$

where C_{it} is the observed annual consumption. Thus, positive percentage errors in Table 51 imply that our regression parameters produced over-estimates of demand.

The most important thing to note about Table 51 is the extent to which we were or were not able to predict realized demand (actual consumption) as measured by the percentage error of each projection. In these terms it appears to be possible to divide the systems into three groups: three (11, 24, 27) show excellent accuracy in prediction with no errors over 6 percent in any year; four others (16, 19, 29, 42) show generally good results with most errors on the order of 10 percent and the largest error

TABLE 51. TEST OF DEMAND PROJECTIONS

System	1963 $\hat{P}$	1963 95% confidence interval	1963 $\frac{C}{N.365}$	1963 % Error	1964 $\hat{P}$	1964 95% confidence interval	1964 $\frac{C}{N.365}$	1964 % Error
11	116.5	121.1 – 111.9	113.7	+ 2.4	120.3	126.7 – 113.9	116.9	+2.8
16	136.2	147.7 – 134.7	127.0	+6.8	143.7	157.7 – 129.7	136.4	+5.1
19	138.4	147.8 – 129.0	130.7	+5.6	143.0	155.7 – 130.3	133.9	+6.4
20	148.0	161.7 – 134.3	141.2	+4.6	153.3	169.8 – 136.8	134.7	+12.1
21	127.5	144.1 – 110.9	141.4	−10.9	111.5	144.7 – 78.3	142.1	−27.4
24	116.7	124.4 – 109.0	121.3	−3.9	120.0	128.9 – 111.1	122.5	−2.1
27	106.5	112.4 – 100.6	106.2	+0.3	106.2	112.0 – 100.4	102.7	+3.3
29	69.7	75.3 – 64.1	79.7	−14.3	70.3	78.3 – 62.3	80.1	−13.9
42	136.3	148.2 – 124.4	124.8	+8.4	140.4	156.0 – 124.8	124.6	+11.2
Average error				−0.1%				−0.3%

TABLE 51—continued

1965				1966				Average town error over 4 years
$\hat{P}$	95% confidence interval	$\frac{C}{N.365}$	% Error	$\hat{P}$	95% confidence interval	$\frac{C}{N.365}$	% Error	
								(*percent*)
124.6	132.6 116.6	118.2	+5.1	126.7	135.5 117.9	119.4	+5.8	+4.0
152.5	171.8 133.2	128.7	+15.6	156.8	180.9 132.7	140.1	+10.6	+9.5
147.8	164.3 131.3	127.9	+13.5	151.1	170.2 132.0	135.2	+10.5	+9.0
159.9	181.3 138.5	121.5	+24.0	163.5	185.5 141.5	117.7	+28.0	+17.2
108.9	182.3 35.5	150.6	−38.2	100.9	178.9 22.9	147.8	−46.5	−30.8
123.3	133.6 113.0	126.7	−2.8	126.1	137.1 115.1	127.7	−1.3	−2.5
110.7	118.9 102.5	113.8	−2.8	114.5	123.7 105.3	120.8	−5.5	−1.2
80.8	100.3 61.3	95.4	−18.1	81.4	99.0 63.8	89.8	−10.3	−14.2
145.7	165.8 125.6	125.1	+14.1	149.0	170.7 127.3	132.4	+11.1	+11.2
			+1.2%				+0.3%	Overall average +0.3%

18.1 percent;[1] two (20 and 21) show great inaccuracy, with maximum error almost 50 percent. In one of these last two classes (system 20) our projections were much too high; this may be partially explained by calls for voluntary restriction of use made by the city's officials at various times during the last three years of the drought. To attribute the entire error to these calls would, however, be at variance with other evidence from the study that voluntary restrictions are notably ineffective.

Although the projections for certain of the individual systems are inaccurate, the average percentage error (across the systems) for each year is very small; in no year is this average error greater than about 1 percent. This suggests that our simple regression/projection equation fails to capture some influences on per capita daily demand which are essentially random and symmetric with respect to an aggregate of communities, one or more of which may influence significantly an individual town's demand over a period. Examples of such influences might be the existence of distributional inadequacies; sudden shifts in the composition of the local industrial sector; or shifts in domestic habits (as in the substitution of disposals for garbage collection).

[1] For this group there is an interesting tendency for projection errors to peak in 1965. This is true for each of the 4 systems. The explanation of this behavior may be that for 3 of these systems, the peak of the drought, as measured by the Palmer Index, occurred in 1965.

APPENDIX B

NOTES ON DATA USED IN THE VARIOUS TESTS

The following notes on data used will be useful to the reader.

Number of Observations. Of the 33 towns for which significant regression relations were found in Chapter 4, 5 were eliminated (as discussed in Chapter 5) because of missing observed-consumption data in the drought years. Of the remaining 28 towns, 4 were eliminated because they either had essentially unlimited safe yield or no safe-yield estimates at all. Four additional towns depended entirely on groundwater supplies; here the concept of safe yield did not apply, and there was no reason to expect the model itself, based on precipitation as a surrogate for streamflows, to apply. This same principle was applied to systems with combinations of ground- and surface-water sources, except where the latter provided a very high percentage of the total system supply. This resulted in the elimination of three additional systems. Finally, two systems were not used in this shortage/adequacy analysis because in the previously reported analysis of "adequate" systems (see Appendix A above—"adequate" systems were those reported as adequate by their managers and not requiring restrictions or emergency supplies at any time during the drought) our projections of demand had appeared to perform very badly. For these two "adequate" systems our projections indicated shortages in *every* drought year of between 5 and 30 percent. The final sample of towns used in the regression analysis was thus reduced to 15.

Calculation of Shortages. Shortages were calculated from the formula given in Chapter 5, Equation 5-4. Figures on emergency supply were obtained from water system managers in the 5 towns which resorted to emergency supplies. In all these instances, the equipment required was loaned to the towns by the Massachusetts Civil Defense organization, and

TABLE 52. CUMULATIVE RAINFALL DEVIATIONS AND α'_t

	α'_t				
	1911	1963	1964	1965	1966
Cumulation period	(1908–11)	(1960–63)	(1961–64)	(1962–65)	(1963–66)
Cumulated deficiency (average for state) (*inches*)	−23.0	−8.3	−28.3	−30.1	−32.7
Calculated α'_t from Equation (6–10)	1	2.77	0.81	0.76	0.70

Note: For each town in each year, the appropriate value of α'_t was subtracted from the estimated actual P/Y ratio to find the proper independent variable for the regression Equation (7-5).

in return the agency required reports of water pumped and general comments on the need for the water and the uses to which it was put. Investigation of these records and discussion with responsible officials at the agency indicated that 3 of these towns in certain drought years used some fraction of their emergency pumpage, not to meet current demands but to refill reservoirs drawn down during the earlier dry years. Use of the total emergency pumping figures for these towns and years would then overstate the size of the current shortage. Unfortunately, the records did not allow an allocation of the pumping figures between these two uses. Our strategy was to reduce the recorded emergency pumpage by amounts which, though basically arbitrary, were based on the comments of the Civil Defense staff and on those recorded in the reports submitted by the towns.

The projected demand figures were calculated as described above. Observed consumption figures were obtained from the records of the towns themselves and those of the Massachusetts Department of Public Health.

Safe Yields. These were obtained, where available, from the system managers in the interviews held with them. In some cases, changes in system safe yields came "on stream" during the drought. Our data on the times of such changes were, again, taken from the interviews.

Calculation of α'_t from Precipitation Data. For each year of the drought (1963–66), the 4-year cumulative rainfall deviations from one station in each of the three Massachusetts climatic divisions (western, central, and coastal) were averaged. The resulting state average deviation was divided into a similar average for the 1908–1911 period.[1] The results of these computations are summarized in Table 52.

[1] It is unfortunate, given what we have observed of the differences in drought timing, that it was not possible to deal with each section of the state separately. This would, however, have resulted in very small samples for each of the regional regressions.

APPENDIX C

DROUGHT ADJUSTMENTS BY THE METROPOLITAN DISTRICT COMMISSION

The Metropolitan District Commission (MDC) is an independent authority created by the Commonwealth of Massachusetts. We present the MDC experience because it affords an opportunity to see the effectiveness of partial restrictions on water use. During most of the 4-year drought, there were no restrictions on water use for MDC customers. However, during a 5-week period late in the summer of 1965, certain restrictions were in force. After a call for voluntary reductions in water consumption failed to produce any decrease in use, mandatory restrictions on "non-essential use of water" were proclaimed by the Massachusetts Department of Public Health upon recommendation of the MDC commissioner.[1] The restrictions were not enforced by MDC, but by the communities which it serves under threat of water cutoff for noncompliance. The ban prohibited the use of water for watering lawns and gardens; for washing cars, driveways, sidewalks, and streets; and for use in non-recirculating air-conditioning equipment, swimming pools, wading pools, and fountains.[2]

The emergency restrictions were proclaimed, not because of a shortage of water *per se* but because of inadequacies in the water distribution system combined with unusually high demand resulting from the dry weather. Wachusett Aqueduct, which carries water from Wachusett Reservoir to MDC customers, has a maximum capacity of 290 million gallons per day (mgd). On several days during the month prior to initiation of the ban, water use in the district exceeded 330 million gallons, with a peak day's consumption of 380 million gallons.[3] The excess of demand over aqueduct

[1] Office of Public Information, press release, August 11, 1965.
[2] *Ibid.*
[3] *Ibid.*

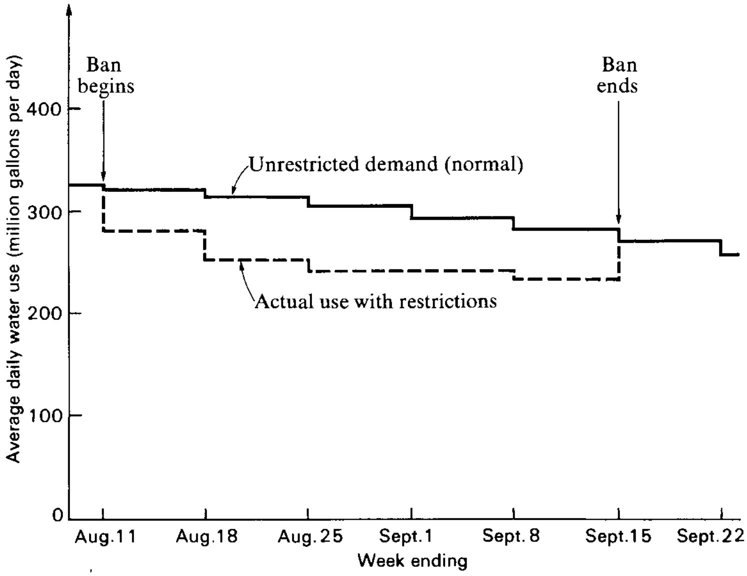

Figure 28. Impact of restrictions on water use by Metropolitan District Commission customers, August 4–September 22, 1965.

capacity had to be drawn from Sudbury Reservoir, which has only a limited storage capacity. The ban was proclaimed, therefore, to bring daily demand down to a level which could be supplied by Wachusett Aqueduct.[4]

How well did the ban work? The immediate purpose of the ban was achieved within the first week, when water use dropped to 280 mgd. (It dropped further in subsequent weeks and did not again rise above 280 mgd during 1965.)[5] The reduction in average consumption amounted to 40–65 mgd, with an average of 55 mgd. For the 5 weeks during which the ban was in force, a reduction in use of about 2 billion gallons total was achieved. This amounts to 2 percent of total use during 1965, or about 0.5 percent of total use during the 4-year drought period.

On the basis of results achieved by the 1965 water ban it appears that a reduction of perhaps 20 percent in water use during the months of peak demand might be expected under such restrictions. This would probably mean a reduction of no more than 10 percent in average yearly consumption.

[4] The Wachusett Aqueduct bottleneck has since been circumvented by construction of the Wachusett-Marlborough Tunnel completed a year later, which should prevent the recurrence of a similar situation.

[5] Unpublished MDC records.

Additional water was made available by MDC to communities it does not normally supply. Such emergency aid was given to the second and third largest cities in Massachusetts—Springfield and Worcester—neither of which is normally supplied by MDC. Other communities, only partially supplied by MDC, managed to avoid problems by taking larger than normal amounts from MDC during the drought. In both cases, the MDC system was subjected to greater stress because accelerated demand increases came at the time that supplies were most critical. The use of MDC water is, for these communities, a relatively painless way of coping with drought emergencies.

APPENDIX D

MEASUREMENT OF BUSINESS LOSSES FROM SHUTDOWN

Consider the case of a firm forced to shut down for a short period by a lack of water. We apply the same principle as in the test—that losses equal gross benefits less costs avoided—but we find a somewhat more complex and subtle problem because of the nature of the firm and conflict between the firm's view and that of society as a whole.

Our first task is the measurement of gross benefits. In this we are assisted by a theorem which states that a firm's willingness to pay, under the usual competitive assumptions, for w_o units of water as an input, can be measured either as:

1. The area up to w_o under the firm's derived demand curve for water as an input, or
2. The difference between the gross value of output using w_o (and the associated optimal input combination) and the (true opportunity) cost of all other inputs involved.[1]

As stated, this theorem seems to give our measure of gross benefits. We must, however, note that the theorem is applicable to the longest-run, to system design, when everything is assumed adjustable. That is, we implicitly assumed in stating the theorem that all inputs other than water may be hired at any required level and that these inputs are mobile between employments. In brief, the assumption was that all non-water input costs were avoidable by the firm and society. This is clearly not the case when we are dealing with temporary shutdown due to shortage. A great many costs are fixed for the firm in the short run, and we will maintain that even more

[1] See Arthur Maass et al., *The Design of Water Resource Systems* (Cambridge: Harvard University Press, 1962), p. 27. The proposition can easily be shown to be true for simple examples such as a two-factor, decreasing-returns, Cobb-Douglas production function.

are fixed for society. These considerations increase the losses from shutdown relative to the statement of the theorem by reducing the amount of cost deducted from the gross value of production lost.

In the short run, the firm will be able to avoid a large part of its variable costs when it shuts down. Wages of most workers, raw materials, housekeeping, and most process-energy requirements are among the most important of such avoidable costs.[2] Thus, from the firm's point of view, the (say daily) losses from the drought are roughly:

$$pY - V_C - P_w w_o$$

where pY is the value of output per day using w_o units of water and the appropriate combination of other inputs; V_C is the avoided variable-cost total except for water costs; and $P_w w_o$ is the daily water bill not paid.

From society's standpoint, the firm's estimate of its losses is too small, for society recognizes a broader sweep of factor immobility than the firm. We note, first, that fixed costs are clearly not avoidable on the part of the firm. Neither are they for society, for the resources, especially capital and entrepreneurship, represented by the fixed payments, are not free in the short run to migrate to alternative employment.[3] Society is paying an opportunity cost for their employment by the firm; this opportunity cost does not vanish merely because of shutdown. This consequence of immobility is recognized by accepting the firm's view of fixed costs as unavoidable. The problem of immobility, however, applies also to certain inputs under variable costs—most importantly to labor. In the short run, the labor force of a firm is very largely immobile, and society must take into account the opportunity cost of this arrangement in estimating costs avoided. Since the impact of odd jobs, expanded moonlighting, etc., is undoubtedly quite small, it seems reasonable to amend the firm's estimate of its losses, in moving to the social view, by subtracting from costs avoided (adding to losses) the value of wages per day. Thus the firm's daily losses corrected for the social cost of labor immobility would be:

$$pY - (V_C - WL) - P_w w_o$$

where WL is the wage bill per day.

If the firm is a self-supplier this expression must be corrected directly for the daily avoided costs of water supply. If it buys from the city we may begin by counting the city's daily lost revenue and correcting that for costs avoided.

[2] We here accept some vaguely "normal" definition of variable costs. To be strictly correct, we should probably maintain the tautological nature of the terminology and call only avoidable costs "variable," all others being "fixed" in the run under discussion.

[3] This ignores any rents or other transfers, just as we ignored them under variable costs.

Thus, $P_w w_o - H_A$ = cost to city of interrupted production, where H_A represents the avoided costs of water supply. It is clear that in any case the aggregate social cost per day is:

$$pY - (V_C - WL) - H_A.$$

It is interesting and important to note that $V_C - WL$ is composed of raw material and energy costs and that hence $pY - (V_C - WL)$ is essentially value added by the firm. (In our example, it is value added per day.) Hence, we arrive at the intuitively plausible and appealing result that value-added per day is a good first measure of business losses from shutdown. This, of course, is before any consideration of transferral or deferral of production and, indeed, before taking account of avoided water losses. Notice, however, that for both domestic and industrial losses, the basic measures discussed directly above and in the text of Chapter 9 are valid for all accounting stances above the individual firm or household.

APPENDIX E

LOSSES FROM RESTRICTIONS ON LAWN–SPRINKLING

This appendix describes in some detail the method used to estimate the losses accruing to the domestic sector because of restrictions on household use of water for sprinkling purposes. We remind the reader that the losses accruing to the municipal sector because of these restrictions were estimated separately in Chapter 9 as "Lost Revenue."

It was not feasible to gather data on these losses by the same methods used for the industrial and commercial sectors because, again, of manpower and money constraints on our efforts. Fortunately, Howe and Linaweaver's work[1] on demand functions for domestic water was completed and available in time to be used for an aggregate level attack on the problem. Our approach was to use their demand function for summer sprinkling water applicable to the eastern United States in areas with metered water and public sewers.

$$q_{s,s} = 0.164\, b^{-0.793}(w_s - 0.6r_s)^{2.93}\, P_s^{-1.57} v^{1.45} \qquad \text{(E-1)}$$

Here,

$q_{s,s}$ = average summer sprinkling demand in gallons per day per dwelling unit;
b = irrigable area per dwelling unit;
w_s = summer potential evapotranspiration in inches;
r_s = summer precipitation in inches;
P_s = marginal commodity charge applicable to average summer total rates of use (cents per 1,000 gal);

[1] Charles W. Howe and F. P. Linaweaver, Jr., "The Impact of Price on Residential Water Demand and Its Relation to System Design and Price Structure," *Water Resources Research*, I (1965), 13–32.

v = market value of dwelling unit in $1,000.[2]

We chose values for the parameters b and w_s from the table provided by Howe and Linaweaver (Table 2, p. 8). In particular, we used the average values for these parameters reported for the 11 eastern areas used in estimating the demand equation.

$$b = 0.147 \text{ acres per dwelling unit}$$
$$w_s = 16.84 \text{ inches per acre}$$

The average value of r_s given by this same table is 10.96 inches per acre. We use 5.00 in/ac in our calculations to reflect the significantly lower rainfalls of drought summers, without aspiring to greater accuracy for this parameter than for the others. Hence $(w_s - 0.6r_s)$ equals 13.84 in/ac. Under average rainfall conditions, $(w_s - 0.6r_s) = 10.26$ in/ac.

Values of v appropriate to each town were calculated from data in the 1960 *Census of Housing*, vol. I, part 4, especially Table 21. v was taken to be a weighted average house value for the town, where the numbers averaged were the midpoints of the various value intervals reported in the table, and the weights were the number of dwellings in the town in each interval. P_s was taken to be the first block rate; that is, the one applicable to lowest total use, since this is invariably the one applicable to domestic decisions about sprinkling. (See Table 53.)

In one significant respect, we depart from the general procedures outlined in the text. That is, for the sprinkling losses of Braintree and Fitchburg, we include in Table 22 the average of the 1965 and 1966 losses. This stretching of our self-imposed rule to use only 1966 losses for these 2 towns was undertaken with the aim of adequately capturing domestic sector sacrifices. (Many of the early critical comments received in connection with this study suggested that we were somehow "missing" some of the drought's domestic sector impact.) To the extent that one of our central findings is the surprisingly small amount of loss (after correction), this policy is distinctly conservative.

As is argued in the text of Chapter 9, the calculation of sprinkling losses in situations in which all sprinkling is forbidden is straightforward. The integral under the given demand curve up to the quantity implied by the price existing before the prohibition will be the measure of total willingness to pay.[3] The net loss to the domestic user will be this area corrected for

[2] A different form for this demand function was reported by Howe in "Water Pricing in Residential Areas," *Journal of the American Water Works Association*, LX (1968), 497–501.

[3] As we note in the text, we assume that water is a small enough factor in the budget that we may use the simple consumer surplus argument.

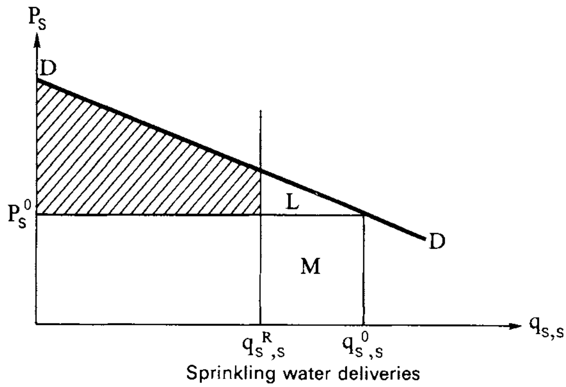

Figure 29. Losses from a partial ban on lawn-sprinkling.

water bills avoided (or consumer surplus). In our present framework, the measure which emerges is in terms of per-dwelling-unit per-day and must be multiplied by the number of dwelling units in the town and the number of days "per summer" to arrive at a total annual loss for the town. This simple situation applied in Fitchburg in 1965, when no sprinkling at all was allowed.

An additional complication is, however, introduced when, as in Braintree in 1965 and 1966, sprinkling is not completely forbidden but only restricted in time. For example, in 1965, Braintree allowed only 1 hour of sprinkling each day; in 1966, this was increased to 2 hours per day. Now, the situation before and after such a restriction may be represented as in Figure 29, where the time restriction appears as a limit on the quantity which can be purchased even though the original price schedule remains in effect. The new net benefit to the consumer is the *shaded area* under the demand curve above the existing price and out to the allowed quantity. The loss *to the consumer* of the restriction is the area L; the gross loss is $L + M$, of which M is borne by the town. The central problem under this situation is, then, to estimate the quantity $q^R_{s,s}$, the amount of which will be used in the restricted time at the existing price.

Our method here was to estimate on the basis of casual empiricism that the average garden use of water would be at the rate of 1 gpm or 60 gph.[4]

[4] This estimate of sprinkling rate is low by comparison with those estimated in the "Final and Summary Report on Phase One of the Residential Water Use Research Project," John C. Geyer, Jerome B. Wolff, F. P. Linaweaver, Jr. (Johns Hopkins, Dept. of Sanitary Engineering and Water Resources, October 1963), p. 10. The rates estimated in that study ranged from 2.90 to 6.19 gpm. Our use of the lower rate is conservative in that it tends to increase the size of losses by decreasing the amount available under restrictions.

Then the use one *could* get per day in Braintree in 1965 was 60 gallons (assuming one hose per house); in 1966, 120 gal. The loss is established by first comparing the possible use, with the desired demand as given by the demand equation. Clearly, if the possible use is greater than the desired, there will be no loss from the restriction (ignoring any psychic losses from not being able to water whenever one wished, etc.).

The Pittsfield situation was, on its face, similar to that of Braintree. Here the restrictions put in effect in 1964 and 1965 allowed 12 hours of sprinkling every other day. But Pittsfield sells domestic water on an annual flat-rate basis, and so the measurement of losses from these restrictions did not seem at all simple; especially since Howe and Linaweaver stress that their results show that the demand for water under a flat-rate is not the same as that implied by a zero price at the margin in the regular demand function. In searching for a measure of willingness to pay for Pittsfield, we came across the information that a very similar city in western Massachusetts, also flat-rate, sells a sprinkling *privilege* exactly equivalent to the Pittsfield restrictions (12 hours every other day). This privilege costs $34.00 per season and is subscribed to by roughly 10 percent of the system's customers who have a need for outside use.[5] It seemed reasonable to conclude that a restriction equivalent in severity to a rather expensive privilege could hardly have resulted in economic losses. Accordingly, we estimated that there were no domestic-sector losses from sprinkling restrictions in Pittsfield. As already noted, no revenue would have been lost in any case, since domestic use is flat-rate.

The actual calculations for Braintree and Fitchburg involved, first, finding the desired quantity of sprinkling water (per dwelling unit per day) in each town at the existing price ($q_{s,s}^{o}$). Then the demand equation was solved for its inverse (p as a function of q), and this was integrated up to $q_{s,s}^{o}$. The form of the inverse is such that the lower limit could not be taken as zero; we chose for simplicity to use 1 as the lower limit of integration. The subsequent corrections to this total willingness to pay estimate to cover the Braintree and Fitchburg examples are straightforward. To obtain annual totals for the towns, 120 days was used for the length of the sprinkling season. The previously calculated number of owner-occupied dwelling units from the *Census of Housing* was used for each town as an estimate of the number of dwelling units affected by the restrictions.

It is interesting to note that the estimates for both towns of total annual household willingness to pay for sprinkling water under average rainfall

[5] The normal sprinkling privilege is 2 hours every other day and costs $8.75 per year. Compare with the 1966 Braintree restriction.

The reason for this rationing of sprinkler time is the distributional inadequacy of this particular system, rather than any shortage of water to deliver.

conditions are quite close to the $8.75 normal-privilege price charged by the town referred to above. These "normal" estimates of willingness to pay are included in Table 53 along with the loss estimates for Braintree and Fitchburg in 1965 and 1966 under the assumed summer rainfall of 5 inches.

TABLE 53. DOMESTIC LOSSES FROM SPRINKLING RESTRICTIONS

	Braintree	Fitchburg
1. Number of owner-occupied dwellings	6,806	4,173
2. V = average value per dwelling ($1,000)	15.4	12.3
3. P_s = price of domestic water (¢/1,000 gal.)	47	33
4. $q_s,^o{}_s$ = desired quantity (gpd/dwelling unit)		
a. under average rainfall	85.9	108.4
b. under 5 inches of summer rain	206.5	260.6
5. Willingness to pay for the desired quantity ($/year/dwelling unit)		
a. under average rainfall	$10.70	$9.70
b. under 5 inches of summer rainfall	$27.43	$24.64
6. Total annual willingness to pay—assuming 5 inches of summer rainfall	$186,700	$101,800
7. Allowed use of water (gpd/dwelling unit)		
1965	60	0
1966	120	no restrictions
8. 1965 loss calculation:		
a. TWTP (Item 6 above)	$186,700	$101,800
b. TWTP for 60 gpd	$107,800	not applicable
c. Market value of difference between desired and allowed purchases (*M* in Figure 29)	$56,200	not applicable
d. Market value of desired purchases	not applicable	$43,000
Losses = *a-b-c-d* (as applicable)	$22,700	$58,800
9. 1966 loss calculation:		no loss
a. TWTP (Item 6 above)	$186,700	
b. TWTP for 120 gpd	$147,700	
c. Market value of difference between desired and allowed purchases (*M* in Figure 29)	$33,200	
Losses = *a-b-c* (*L* in Figure 29)	$5,800	
10. Losses included in Table 22 et seq.	$14,200	$29,400 (see text)

APPENDIX F

CORRECTIONS FOR INVESTMENT RETURNS

First, we must distinguish between three aspects of a firm's "use" of water as an input. Usually, a certain quantity of water per unit of time is required to *flow through* the plant. For example, given information about plant efficiency, load factor, condenser design, etc., for a thermal electric plant we can predict the total quantity of water required to pass through the condensers per year. It is not, however, necessary that the plant *withdraw* from its source(s) an amount equal to the total flow-through use. Water use technology offers the manager a range of water use patterns from once-through use to a very high degree of recirculation and consequently low withdrawal requirement relative to flow-through use. (In the thermal-electric case, the manager may decide to erect cooling towers through which the condenser cooling water is recirculated to reduce its temperature by evaporation. The water may then be reused in the condenser.) In most uses of water, some of the water withdrawn is *lost*. (Evaporative losses in the cooling towers or in the stream warmed by the condenser water effluent are the relevant losses in our thermal electric example.)[1]

Within this framework we can discuss the effect of investment projects on annual water costs in very simple terms. In the case of wells, what is at stake is usually the substitution of one source for another. That is, firms and individuals drilling wells are most often interested in substituting their own for city water with its attendant restrictions. No change in total use, withdrawals, or losses is involved.[2] The changes are only in the sources of

[1] Blair T. Bower, "The Economics of Industrial Water Utilization" in *Water Research*, Allen V. Kneese and Stephen C. Smith, eds. (Baltimore: Johns Hopkins Press for Resources for the Future, 1966).

[2] Note that wells might also be dug to replace surface water self-supply or to augment existing wells or surface supplies.

withdrawals and consequently in their price. For example, let us define the following variables:

T(1,000 gal) = total annual withdrawals before adjustment

L($ per 1,000 gal) = marginal block rate for city water

ν = fraction of required withdrawals supplied by the new well

V_w($ per 1,000 gal) = variable costs of well water

F_w($ per 1,000 gal) = fixed costs of well water (annual capital charge; we ignore other possible fixed charges)

We also define

$$Q = \frac{(1 + r)^N - 1}{r(1 + r)^N}$$ = present worth factor for a constant stream of future earnings. Here r is the rate of discount and N is the length of the stream (project life).

Then,

$$\frac{1}{Q} = \frac{r(1 + r)^N}{(1 + r)^N - 1}$$ = capital recovery factor,

the annual charge which, if the discount rate is r and the life of the project N years, will produce a constant-cost stream of present value K, the original capital investment. Then in our notation

$$F_w = (K_w/Q)\cdot(1/\nu T) \tag{F-1}$$

where K_w is the capital cost of the well.
Further define:

$$S_w = V_w + F_w = \text{total annual cost per 1,000 gal of well water.} \tag{F-2}$$

Now, before the adjustment, the total annual cost of water to the firm was: TL (approximately only, because of the usual block rate structure). After the adjustment, the total annual cost is:

$$\nu TS_w + (1 - \nu)\,TL \tag{F-3}$$

For there to exist a net savings in water costs during each year of the well's life it must be true that:

$$TL - [\nu TS_w + (1 - \nu)\,TL] > 0 \tag{F-4}$$

or that L be greater than S_w. This same condition will insure that the project's present value will be positive, for that requirement is that,

$$Q[TL - (\nu TS_w + [1 - \nu]\, TL)] > 0 \tag{F-5}$$

and $Q > 0$ necessarily if $1 > r > 0$. Expressed in terms of variable costs, the requirement is that

$$Q[TL - (\nu TV_w + [1 - \nu]\, TL)] > K_w.\text{[3]} \tag{F-6}$$

The formulae required for the discussion of recirculation projects are somewhat more complex but the basic ideas are equally straightforward. Recalling the variables defined before, but replacing the subscript w with R, let us consider a particularly simple example. We assume that before adjustment the firm's T thousand gallons of withdrawals from city water represent also its flow-through use. (The plant's use of water is once through.) As above, costs before adjustment are approximately TL. Now, we assume that the firm installs a cooling tower with capital cost K_R capable of recirculating Y thousand gallons per year. The relationship between Y and K_R depends on the cooling requirements, climatic conditions, and hours of operation.[4] Of the total recirculation volume some fraction, m, will have to be made up from city water due to evaporation and blow-down losses. Defining S_R to be average total cost per 1,000-gallons exclusive of make up costs, we may write total water costs after the adjustment as either:

$$(S_R + mL)\, Y + (T - Y)\, L, \tag{F-7}$$

or

$$S_R Y + (T - Y + mY)\, L. \tag{F-8}$$

For the adjustment to produce a stream of benefits (net water cost savings) over its life, it must be true that:

$$TL - [S_R Y + (T - Y + mY)\, L] > 0, \tag{F-9}$$

or that,

$$L(1 - m) > S_R. \tag{F-10}$$

For example, Equation F-10 says that for a water-cost saving to exist with 10 percent make-up feed, the average total cost of the recirculated water

[3] The equivalence of Expressions F-5 and F-6 is easily proved by substituting from F-1 and F-2 into F-5.

[4] See, for example, Paul Cootner and George O. G. Löf, *Water Demand for Steam Electric Generation* (Baltimore: Johns Hopkins Press, 1965), Ch. 6.

(exclusive of make-up costs) must be less than 90 percent of the local block rate. (By the same reasoning used above, this is sufficient for the project to have a positive present value.) Alternatively, in terms of variable costs we have the requirements,

$$QY[L - mL - V_R] > K_R. \tag{F-11}$$

Again, the equivalence of the statements in terms of S_R and V_R follows directly from the definition of S_R.

APPENDIX G

PRODUCTION LOSSES AND NET EARNINGS REMITTALS UNDER VARIOUS ACCOUNTING STANCES

If we assume that production is "lost" at one time or place but made up at some other time or place, our estimate of the extent of factor employment at the latter will be directly relevant to our assessment of the cost this transfer represents to society. If full employment holds for the time or place to which the shift occurs, then the cost of increasing production is measured by the marginal cost of leisure foregone (wages), of increased depreciation (if capital is also fully employed), and of the cost of spreading managerial ability thinner. Interest and rent charges would not increase. Discussion of profits in this context is made very difficult by the lack of agreement among economists as to just what profits are. If profits are a return to managerial ability, our comments above apply. If, however, profits are a return for the bearing of uncertainty, the making of decisions, or the introduction of innovations, it is more difficult to see just what the impact of transfer or deferral would be. One might adopt an assumption tying the supply of the relevant function to the level of employment of other factors. Thus, if full employment were the rule, there will also be some social cost of drawing out a bigger supply of the risk-bearing or decision-making function.

Given this framework of principle, we must decide what corrections to make in practice. We first note that the important variable here is the extent of labor unemployment. We may think of capital as nearly always underemployed in a social sense so long as one shift is the general rule. Alternatively we can take the view that whether or not it is said to be fully employed is irrelevant, since we can intensify its use with a negligible increase in social cost. Similarly, we may ignore the other factors in worrying about the impact of unemployment. We further agree to consider only two cases called "unemployment" and "full employment" and to assume that at full employment a given amount of "lost" value added can be replaced at a different time or place at a social cost approximately equal to the money costs of hiring the required labor (or persuading the required labor to give up leisure at the margin). We may then say that that part of the lost value-added attributable to wages represents the social cost of its replacement at a different time or place under full employment. To this basic cost must be added the increased transport charge in the case of spatial transfer or the cost of waiting where a temporal shift is involved. Thus, we summarize in Table 54 the costs if under a given accounting stance we determine that transfer or deferral of production losses must have occurred.

TABLE 54. COSTS OF "LOST" PRODUCTION UNDER TRANSFERRAL OR DEFERRAL

Transfer		Defer	
Full employment	Unemployment	Full employment	Unemployment
Wages and transport costs	Transport costs	Wages and waiting costs	Waiting costs

In translating this scheme into use with our data, we follow work done by Kates in the field of flood losses.[1] In this work Kates reached, by a rather different line of reasoning, conclusions essentially the same as we have set out above. In particular, we make use of his approximation of transport and waiting (he uses a different term here) costs of 2 percent of the gross value of production "lost." Not only does this figure appear intuitively reasonable, but our use of it is a step in the direction of comparability of losses from various natural "disasters." Potentially, then, even with our limited consideration of unemployment possibilities, we have an infinity of situations to consider, depending on the degree of transferral or deferral assumed under each of the accounting stances. In order, again, to limit our task to reasonable proportions, we originally considered the following situations:

[1] See Robert W. Kates, *Industrial Flood Losses: Damage Estimation in the Lehigh Valley*, Department of Geography Research Paper No. 98. (Chicago: University of Chicago Press, 1965), pp. 47-57.

At Each Discount Rate—8 percent and 20 percent

Local Stance	a. no transfer/defer
	b. 50 percent transfer/defer—full employment
Regional Stance	c. 25 percent transfer/defer—unemployment
	d. 50 percent transfer/defer—unemployment
National Stance	e. 75 percent transfer/defer—full employment
	f. 100 percent transfer/defer—unemployment

The same principles apply and the same methods are used in correcting retail-sector business losses as for the production losses in the industrial sector. The same set of situations is considered in order to preserve consistency even though it seems clear that deferral is almost certainly the rule for these losses given sufficient time.

The second basic question we must deal with here is that of the method of allowing for the remitting of net returns to central headquarters by the divisions of national firms represented in our survey. The impact of this correction will be to decrease the positive present value (perhaps even driving it negative) associated with the aggregate of industrial investments in the local and regional accounts. This correction is made under the following assumptions:

1. That 95 percent of net earnings of national firms are drained out of the local area.
2. That 80 percent of net earnings of such firms are drained out of the region (the state).
3. That we may ignore international transactions and treat 100 percent of such earnings as accruing to the nation's account.

In actually reporting the results of our calculations, we have confined ourselves to 4 of the 12 sets originally considered—the local 0 percent deferred, and national, 100 percent deferred accounts at each discount rate. This selection has the effect of keeping down the volume of information competing for the reader's attention; it is, however, justified as discussed in the text, on the basis of the relevance of two of the chosen accounts for key decision-making levels. The other two are presented primarily to show the effect of changing interest rates.

APPENDIX H

METHODS OF SOLUTION

Initially, solutions to the model under various combinations of parameter values were sought using a simple search technique on the time-sharing computer. This technique was chosen after we had first gone over the response surface fairly carefully under one set of parameters and found apparently only one optimum vector. In the general search procedure, an initial vector $\{T_1^1, T_2^1, S_0^1, S_1^1, S_2^1\}$ was chosen. A program was written which looked at the 125 vectors formed by considering $x_i^1 - \Delta x_i^1$, $x_i^1 + \Delta x_i^1$ for each element, x_i^1, of the initial vector. The program chose that vector giving the minimum value to total cost. The initial T_i's were always chosen as follows:

$$T_1^1 = 20; \quad \Delta T_1^1 = 10$$

$$T_2^1 = 40; \quad \Delta T_2^1 = 10.$$

The s_i^1's were chosen differently depending on the total increment which had to be provided from Equation 14-13, which equals $D_o(e^{60} - 1)$. In general, it had to be true that for $\{S_0^1 + \Delta S_0^1, S_1^1 + \Delta S_1^1, S_2^1 + \Delta S_2^1\}$, s_{60} was non-negative.

The vector giving minimum cost in the initial search was chosen as the base vector for a second round in which the intervals used were reduced to ½ those used in the first run ($\Delta x_i^2 = \frac{1}{2}\Delta x_i^1$). This procedure was repeated until the resulting reduction in the total costs was less than some $\epsilon > 0$.

The results obtained from this technique were encouraging in that the changes in optimal solution values of the vector $\{x_i\}$ with changes in the important parameters (particularly ρ, y, and z) were generally intuitively acceptable. It seemed, however, that a more efficient method of taking into account the possibility of multiple local optima was needed. Accordingly, it was decided to attempt to use a non-linear programming package based on the Zoutendijk method of feasible directions.

The basic outline of the Zoutendijk method is well known. It is applicable to the problem of minimizing a function, $f(x)$, subject to a set of linear constraints, $Ax = b$, defining a closed, convex region; and to non-negativity constraints, $x > 0$. If $f(x)$ is strictly convex, the method will find the global optimum.

There are two essential subproblems involved at each step in the iterative procedure: the direction-finding problem and the step-size problem. The process is initiated at some feasible vector (one satisfying the set of constraints). In the direction-finding problem the method attempts to find a direction in which to move away from the initial (or subsequent) vector. This direction should be as close as possible (in some sense) to the negative of the gradient of $f(x)$ at that initial (or subsequent) vector, subject to the restriction that some movement may be made in that direction without leaving the feasible region. The actual choice of the direction of movement involves specification of a normalization rule which protects us from choosing directions with infinite components.

The step-size problem involves the choice of the best distance to move along the feasible direction. This problem may be solved by a search technique or through a sub-program optimizing the step-length for the given gradient information.[1]

Because our objective function is non-convex, we must concern ourselves with the possibility that the solutions obtained are merely local optima. We attempted to deal with this problem by using three different starting vectors for each of our solution runs, hoping that the same solution vector would be found from each starting point. In a six-dimensional space, of course, it may be argued that using only three vectors does not even begin to answer the question; that there are just too many nooks and crannies which will be ignored unless we use a large number of different initial vectors, perhaps choosing them at random.[2] Our problem is, however, simple enough that we may hope to be able to make use of economic intuition in choosing a small set of vectors spanning a large relevant portion of the space. In particular, the vectors we chose to use are shown in Table 55.[3]

[1] For a fuller explanation of the methods involved, see Douglas Shier, "The Zoutendijk Method: A Computational Analysis," unpublished undergraduate honors thesis for the Department of Applied Mathematics at Harvard College, May 1968.

[2] For example, we could use the method developed by Peter Rogers. See his "Random Methods for Non-convex Programming," Doctoral dissertation presented in 1966 to the Division of Engineering and Applied Physics at Harvard University.

[3] Because of the form of the partial derivatives of the objective function, the initial vector could not be chosen with any zero capacity increments (except, of course, for the

TABLE 55. STARTING VECTORS

Elements	1	2	3
T_1	5	5	29
T_2	10	55	30
s_0	$D_0(e^{60\alpha} - 1) - 3$	$\frac{D_0(e^{60\alpha} - 1)}{4}$	1.0000
s_1	1.0000	$\frac{D_0(e^{60\alpha} - 1)}{4}$	$\frac{D_0(e^{60\alpha} - 1)}{2} - 1$
s_2	1.0000	$\frac{D_0(e^{60\alpha} - 1)}{4}$	$\frac{D_0(e^{60\alpha} - 1)}{2} - 1$
s_{60}	1.0000	$\frac{D_0(e^{60\alpha} - 1)}{4}$	1.0000

Thus, the differences between the vectors are large in terms of the timing of the major part of the required capacity expansion. In Vector 1, nearly all the total addition to safe yield is constructed early in the planning period. In Vector 3, a large block of capacity is added at the middle of the period; while under Vector 2, about half is built early, half rather late.

The rules for the termination of the program, that is, for identification of an optimum, were the following:

1. The program terminated if no feasible direction of movement could be found; that is, if either the solution to the direction-finding problem was a zero vector, or if the only feasible step length in the best direction was zero.

2. The program also terminated when along a feasible direction the optimal step length resulted in a reduction in the value of the objective function of less than 10^{-7}.

3. Finally, the program terminated if accumulated round-off errors in the interim solution vector resulted in that vector becoming infeasible (violating the constraint) by more than 10^{-6}.

COMPARISON OF PROGRAMMING AND SEARCH RESULTS

We present in Table 56 the results of comparing the total costs of the "optimal" solutions found by the programming algorithm with those

slack element). In principle, such an element would make the problem impossible because terms in one or more of the partials would not be defined. In practice, the computer adopts some approximation, and whether or not a meaningful solution is reached depends on the parameter values being run; for some values, the program is derailed almost immediately, while for others, it may perform as well as when started from a legitimate vector.

discovered using the search technique. These results are presented for 27 combinations of z, y and ρ; no search results were available using $z = 12.0$. Most striking, perhaps, is the closeness of agreement between the two sets of costs. The difference between the two figures is only once larger than 5 percent and is often less than 1 percent. (Small percentage differences in the total costs generally imply considerably larger differences in the elements of the choice vector, but where the cost agreement is particularly close, the optimal paths are also nearly identical.)

TABLE 56. COMPARISON OF SEARCH RESULTS WITH PROGRAMMING SOLUTIONS: TOTAL COSTS

			Minimum Total Cost			
z	y	ρ	Search routine	Programming solution	Lower cost	Percentage difference
5.4	0.88	0.07	$\$1.151 \times 10^6$	$\$1.145 \times 10^6$	Program	0.52
5.4	0.88	0.05	1.729×10^6	1.727×10^6	Program	0.11
5.4	0.88	0.03	2.837×10^6	2.836×10^6	Program	0.03
5.4	0.78	0.07	0.964×10^6	0.980×10^6	Search	1.65
5.4	0.78	0.05	1.419×10^6	1.419×10^6	Program	0
5.4	0.78	0.03	2.235×10^6	2.214×10^6	Program	0.95
5.4	0.68	0.07	0.800×10^6	0.835×10^6	Search	4.37
5.4	0.68	0.05	1.162×10^6	1.159×10^6	Program	0.25
5.4	0.68	0.03	1.729×10^6	1.697×10^6	Program	1.88
4.3	0.88	0.07	1.052×10^6	1.059×10^6	Search	0.66
4.3	0.88	0.05	1.662×10^6	1.655×10^6	Program	0.42
4.3	0.88	0.03	2.794×10^6	2.794×10^6	Program	0
4.3	0.78	0.07	0.895×10^6	0.906×10^6	Search	1.23
4.3	0.78	0.05	1.356×10^6	1.381×10^6	Search	1.84
4.3	0.78	0.03	2.225×10^6	2.203×10^6	Program	1.00
4.3	0.68	0.07	0.756×10^6	0.798×10^6	Search	5.55
4.3	0.68	0.05	1.102×10^6	1.141×10^6	Search	3.54
4.3	0.68	0.03	1.736×10^6	1.712×10^6	Program	1.40
3.2	0.88	0.07	0.914×10^6	0.948×10^6	Search	3.71
3.2	0.88	0.05	1.514×10^6	1.522×10^6	Search	0.52
3.2	0.88	0.03	2.702×10^6	2.711×10^6	Search	0.33
3.2	0.78	0.07	0.798×10^6	0.835×10^6	Search	4.63
3.2	0.78	0.05	1.264×10^6	1.294×10^6	Search	2.73
3.2	0.78	0.03	2.141×10^6	2.147×10^6	Search	0.28
3.2	0.68	0.07	0.692×10^6	0.708×10^6	Search	2.31
3.2	0.68	0.05	1.049×10^6	1.088×10^6	Search	3.72
3.2	0.68	0.03	1.691×10^6	1.708×10^6	Search	1.00

The search method we used is rather simple. For it to arrive successfully at the optimum requires a simple shape for the objective function; otherwise, each time we halve the search distance, we risk missing the area in which the optimum is located. Thus, the fact that this method leads us to solutions either very close to or better than those achieved by the programming algorithm may be seen as evidence that our objective function is, in fact, quite well behaved.

It is also significant that the search method tends to produce the better results for those parameter combinations with which the program had the most trouble. In particular, when z equals 3.2, the search method invariably turns up the lower total cost figure, while when $z = 5.4$, the search solution is lower cost in only two instances. We noted above that the problems the program experienced for low values of z were due to the tendency for zero-capacity increments to appear in the solution vector under those conditions. It is not surprising to find that a technique making relatively large steps in each direction in searching for the optimum is less sensitive to the corner-solution problem, a symptom of the myopia of the methods of the calculus. What is, perhaps, surprising is that the programming solutions do as well as they do. The available evidence suggests that while there are some difficulties in assuring the discovery of the global optimum (particularly when z is small), we may be confident that the actual solutions found do not give rise to total costs more than 5 percent above the best attainable.