# PART IV

# A PLANNING MODEL FOR MUNICIPAL WATER SUPPLY SYSTEMS

**CHAPTER 12**

# EXPECTED ANNUAL LOSSES FROM DROUGHT

Our aim now is to put together a planning model designed to answer questions about the optimal size and timing of increments to the supply capability of municipal systems, a model drawing together the separate threads represented by the study of the physical and economic impact of the 1963–66 drought as reported in Parts I and II. To this end we first demonstrate how we may use probability distributions of climatic events to produce functions relating chosen levels of system inadequacy to expected levels of per capita annual losses from water shortage (drought). If we think in terms of the inventory-control or capacity-expansion models used by other authors,[1] the expected annual loss functions we develop are seen as the penalties attached to "undercapacity," although in our model there is no line between under- and over-capacity as such but only degrees of relative inadequacy as represented by the chosen level of the projected-demand/safe-yield ratio.

In order to make rational decisions about avoiding losses, we need to know the cost of doing so. In our model, this cost will be that of adding increments of safe yield to the water system. In Chapter 13, we describe two independent derivations of such a function. Then, in Chapter 14, we set out the planning model constructed from these building blocks. We discuss the form the model takes when expressed as a nonlinear programming problem and then go on to explain briefly the method used in finding

[1] See for example, H. B. Chenery, "Overcapacity and the Acceleration Principle," *Econometrica*, 20 (1952), 1–28; A. S. Manne, "Capacity Expansion and Probabilistic Growth," *Econometrica*, 29 (1961), 632–49; A. S. Manne, *Investment for Capacity Expansion* (Cambridge: Massachusetts Institute of Technology Press, 1967); and H. A. Thomas, "Capacity Expansion of Public Works," Division of Engineering and Applied Science, Harvard University (mimeo.), 1967.

the solutions. We also summarize the results of applying the planning model to hypothetical water systems under various assumptions about the relevant parameters. First, we assume away, temporarily, the important question of uncertainty in the projections of demand, being initially concerned with exploring the sensitivity of our model to changes in such parameters as the rate of discount, the scaling factor in the safe-yield cost function, and the exponent in the expected-loss function.[2] Finally, we take account of the uncertainty inherent in projections of the rates of growth of population and per capita daily demand averaged over the year. Essentially we ask how much the town can expect to lose by building according to a plan optimal for the "best" estimates of these growth rates, when in fact other growth rates may actually occur.

One final general comment is in order. In constructing a planning model which minimizes the sum of capacity costs and shortage losses in determining optimal patterns of size and timing of additions to safe yield, we must assume that all alternative paths seek to provide the same streams of gross benefits.[3] Specifically, we assume that no price changes are to take place over the planning horizon, and that the demand functions for system water supply are to be independent of the decisions made by the planners. (This neglects any influence that uncertainty of supply itself might have on demand.)[4] Then the benefits which result from meeting demand under any plan are the same. The differences between the plans lie in their capital costs and in the extent to which they may be expected to provide for demand in the face of a variable climate (drought losses). Minimizing the sum of capital costs and expected drought losses gives us the optimal plan under the given demand conditions.

## ESTIMATING EXPECTED-LOSS FUNCTION

We are interested in two sets of expected-loss functions: one corresponding to the a priori model of drought impact; and the other based on the empirical relation between shortage and inadequacy estimated from the actual drought data. (Both relations were discussed in Chapter 7.) It will be seen that the functions implied by these two different views are

[2] This function, as we shall see, is of the form:

$$E(L) = U(D/Y)^z$$

where $E(L)$ is expected annual loss, $D$ is average daily system demand, $Y$ is system safe yield in daily draft terms, and $U$ and $z$ are parameters. We are interested in the model's sensitivity to changes in our estimate of $z$.

[3] Peter O. Steiner, "The Role of Alternative Cost in Project Design and Selection," *Quarterly Journal of Economics*, 79 (1965), 415–30.

[4] See Stephen Turnovsky, "The Responses of Economic Factors to Uncertainty in Supply," in *Models for Regional Water Management*, R. Dorfman, H. Jacoby, and H. Thomas, eds., to be published by Harvard University Press.

themselves dramatically different. Specifically, expected losses rise very much more rapidly with declining system adequacy (rising demand/supply ratio) under the hypothesis that the a priori model is, in fact, an accurate description of the world.[5]

### *The Relation Between Shortage and System Adequacy: A Reminder*

We pause to remind the reader of the two alternative forms of the shortage-adequacy relation growing out of our work in Chapter 7.

*The A Priori Model.*

$$S_{it} = 100[1 - (\alpha_t^*/\alpha_{it})] \tag{12-1}$$

where $\alpha_{it} = D_{it}/Y_{it}$ and $\alpha_t^*$ is the fraction of safe-yield flows available in year $t$.[6]

*The Empirical Model.*

$$S_{it} = \beta[\alpha_{it} - \alpha_t'] \tag{12-2}$$

where $\beta = 20$ and

$$\alpha_t' = \frac{\Delta_o}{\Delta_t} = \frac{\text{Cumulative Precipitation Deviation 1908–11}}{\text{Cumulative Precipitation Deviation (years } t\text{-3 through } t)}$$

and $\alpha_t'$ is the level of $\alpha_{it}$ at which shortages begin to occur.

## PROBABILITY OF SHORTAGES

We wish to find the probability of a shortage of size $\underline{S} \leq S \leq \bar{S}$ occurring in a city served by a system having inadequacy level $\alpha_{it} = I$. This probability clearly will depend on the distribution of climatic events for

[5] It is unfortunate that the full implications of a priori models designed to determine percentage of safe-yield flows available were not realized earlier. Work based on the empirically determined relation between shortage and adequacy consistently suggests that public and managerial concern over drought is exaggerated. The a priori model, however, tends to lend support to high level of this concern, and in the context of the planning model, leads to capacity expansion programs which look very much like patterns which have been criticized as "overbuilding" when they have appeared in the real world. (See Part V.)

[6] Based on the experimentation, referred to earlier, with streamflow and rainfall records, $\alpha_t^*$ has been estimated from the relation:

$$\alpha_t^* = \frac{(\text{cumulated total precipitation years } t\text{-3 to } t)^2}{(\text{cumulated total precipitation years 1908–11})^2}$$

As noted in Chapters 6 and 7, the actual results of this experimentation and of our attempts to test the a priori model were basically disappointing. It still seems, however, to be worth while to present the loss functions based on the a priori model.

that town. The latter distribution will define the probability of an event $A_* \leq \alpha_t^* \leq A^*$ (or $A_1 \leq \alpha_t' \leq A^1$); that is, the probability of a natural event sufficiently severe to cause a shortage in the size interval of interest. For a given $\alpha_{it}$, the larger shortage, $S$, will correspond to the more severe climatic event $A_*$, i.e., the lower percentage of system safe yield available for delivery.[7]

## APPROXIMATION OF EXPECTED LOSSES

Repeated application of the above methods will provide us with the probabilities of shortages in the range $S$ to $S + \Delta S$ percent, for $S = 0, \ldots 50, \ldots$, for any given level of $\alpha$. We chose to calculate these probabilities for $\alpha = 0.35, 0.45, \ldots, 1.45, \ldots$ and to let $\Delta S = 2$ percent. We may then approximate the expected loss corresponding to a particular chosen $\alpha$ by identifying with the interval $S$ to $S + 2$ percent, the per capita annual losses associated with $(S + 1)$ percent. The appropriate loss data may be read from the functions in Figure 19 for the interval up to about 25 percent shortage. For higher shortages, we approximated the per capita annual losses by functions roughly fitted to the lower interval points for each of the curves, local/20 percent and national/8 percent. The expected loss associated with a particular inadequacy ratio, $I$, is then approximated by:

$$E(L) = \sum_{S'=0}^{98} \text{Prob}\,(S' \leq S \leq S' + 2|\alpha = I) \cdot L(S' + 1) \qquad (12\text{-}3)$$

where $L(S' + 1)$ is the per capita annual loss associated with a shortage of size $(S' + 1)$.[8]

[7] For the a priori model, the expression Prob $(\underline{S} \leq S \leq \bar{S}/\alpha = I)$ may be translated into:

$$\text{Prob}\left[100\left(1 - \frac{A^*}{I}\right) \leq S \leq 100\left(1 - \frac{A_*}{I}\right)\right];$$

which is equal to

$$\text{Prob}\left[\frac{I(100 - \bar{S})}{100} \leq \alpha_t^* \leq \frac{I(100 - \underline{S})}{100}\right].$$

This last expression may be evaluated in terms of the relevant distribution of climatic events.

For the empirical relation, the corresponding probability expression is

$$\text{Prob}\left[\left(I - \frac{\bar{S}}{20}\right) \leq \alpha_t' \leq \left(I - \frac{\underline{S}}{20}\right)\right]$$

[8] This expected-loss level is, strictly speaking, a long-run concept; the loss which would, on the average, occur annually in a town which maintained a particular level of the $D/Y$ ratio over a long period. As we use it here, the concept may be considered as a best estimate of the loss likely to occur in a town with a particular $D/Y$ ratio in a particular year.

Concentration on expected value, which seems entirely adequate with respect to the results from the empirical model, may be questioned when dealing with the a priori model because of the rather large losses encountered there.

The results of these calculations are summarized graphically in Figure 20, where we present the expected annual loss functions under both the local/20 percent and the national/8 percent accounting combinations and for both the a priori and empirical models. The expected-loss functions associated with the a priori model rise much more rapidly than those associated with the empirical model. The former are, indeed, almost vertical in the interval above about $\alpha = 1.10$. We shall see that the use of this sort of function has some significant implications for the planning of system expansion.

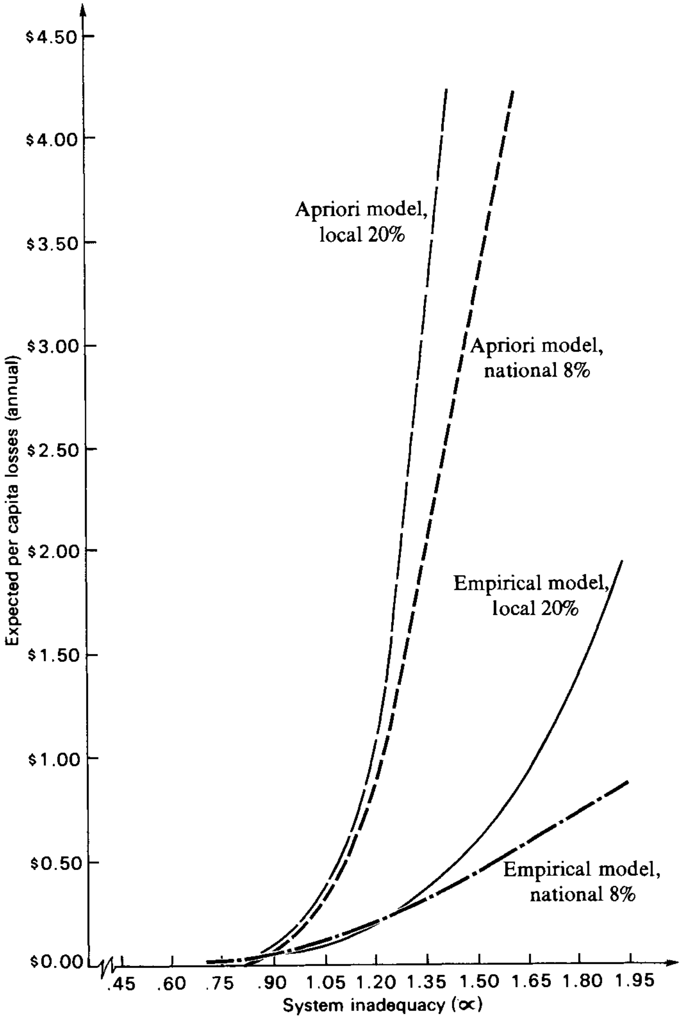


Figure 20. Expected-loss functions.

## ESTIMATION OF THE PARAMETERS OF EXPECTED-LOSS FUNCTIONS

In order to retain the simplest possible loss-function form for use in the planning model, and because it seemed conceptually correct, we fitted to our estimates of expected losses for given $\alpha$, functions of the form:

$$E(L) = U(\alpha)^z \text{ (dollars per capita per year).}$$

The results of this operation are summarized below in Table 35.

TABLE 35. PARAMETERS OF THE EXPECTED-LOSS FUNCTIONS

(*$ per capita per year*)

| | | |
|---|---|---|
| I. | A priori model: | |
| | Local/20 percent account | $E_1(L) = 0.1(\alpha)^{12.0}$ |
| | National/8 percent account | $E_2(L) = 0.1(\alpha)^{11.7}$ |
| II. | Empirical model: (fitting to points above 0.35) | |
| | Local/20 percent account | $E_3(L) = 0.1(\alpha)^{5.4}$ |
| | National/8 percent account | $E_4(L) = 0.1(\alpha)^{4.5}$ |
| III. | Empirical model: (fitting to points above 0.95) | |
| | Local/20 percent account | $E_5(L) = 0.1(\alpha)^{4.8}$ |
| | National/8 percent account | $E_6(L) = 0.1(\alpha)^{3.2}$ |

*Note:* The extraordinary constancy of the estimates of $u$ was caused by the relative constancy of the estimates of expected losses for all stances and models in the interval about $\alpha = 1.00$. These were invariably in the range $0.10 to $0.13.

In our exploration of the sensitivity of the planning model to parameter changes we use four different values of $z$: 3.2, 4.3, 5.4, and 12.0. These effectively cover the range of results presented in Table 35, and it seems likely that they span the "true" values.

Now that we have our functions relating chosen inadequacy levels to expected drought losses, we need only one more bit of information to be able to construct the planning model: an estimate of the cost of improving the level of system adequacy.